MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2018/2019

DIM5068 – MATHEMATICAL TECHNIQUES 2

(For DIT students only)

25 OCTOBER 2018
2.30 p.m. – 4.30 p.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of **2** pages with **4** questions excluded the cover page and Appendix. Key formulae are given in the Appendix.
2. Answer **ALL** questions.
3. Write your answers in the answer booklet provided.
4. All necessary working steps must be shown.

Question 1

a. Differentiate the following functions with respect to x by using **Chain Rule**.

i) $f(x) = \ln(2x^3 - 4x^2 - 4x)$. (5 marks)

ii) $y = -\dfrac{6}{\sqrt[3]{2x^3 + 4x}}$. (6 marks)

b. If $7y^2 - 4x^5 + 2xy^2 = x^4 y$, show that $\dfrac{dy}{dx} = \dfrac{20x^4 - 2y^2 + 4x^3 y}{14y + 4xy - x^4}$. (6 marks)

c. Given $f(x) = x^3 - 6x^2$

i) Find the intervals on which the function is increasing and decreasing. (6 marks)

ii) Identify the function's local extreme values. (2 marks)

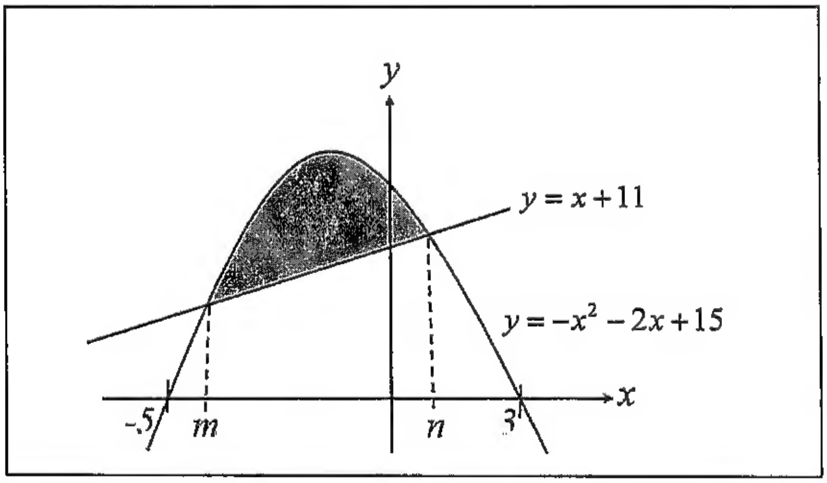
[TOTAL 25 MARKS]

Question 2

a. Use **Substitution Rule** to find $\int (3x^3 - 1)(3x^4 - 4x + 11)^{10}\, dx$. (7 marks)

b. Determine $\int (2x-1)e^{2x-1}dx$ by using the **Integration by Parts.** (7 marks)

c. The diagram below shows the curve of $y = -x^2 - 2x + 15$ and the straight line of $y = x + 11$.



i) Show that the value of $m = -4$ and $n = 1$. (5 marks)

ii) Find the area of the shaded region. (6 marks)

[TOTAL 25 MARKS]

Continued...

Question 3

a. Solve the differential equation, $\frac{dy}{dx} = \frac{7-3x^2+\sec^2 x}{y^3}$ by using **separable method.** (5 marks)

b. Use the **method of integrating factors** to solve the differential equation, $x^5 \frac{dy}{dx} + 3x^4 y = x^9 + x^2 e^x$ given that $y(0) = 100$. (11 marks)

c. Find the **general solution** of non-homogeneous equation, $y''+8y'-33y = 66$ which consists of **complementary solution,** y_c and **particular solution,** y_p. (9 marks)

[TOTAL 25 MARKS]

Question 4

a. Let $\mathbf{a} = 4\mathbf{i} - 2\mathbf{k}$ and $\mathbf{b} = 6\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$

i) Compute $2\mathbf{b} \bullet (-3\mathbf{a})$. (4 marks)

ii) Find the value of x and y if $\mathbf{a} + \mathbf{b} = \langle y+3x, x, 1 \rangle$. (3 marks)

b. Andy wants to sketch a triangular shape. Given the vertices of the triangle $A = (2, 1, 0)$, $B = (3,5, 7)$, and $C = (4, 3, 10)$.

i) Determine $\vec{AB}$ and $\vec{AC}$. (2 marks)

ii) Calculate the cross product of $\vec{AB}$ and $\vec{AC}$. (3 marks)

iii) Compute the total area of the triangle. Round your answer to 2 decimal places. (3 marks)

c. If a line passing through the points (-2, 1, -6) and (0, 4, -2), compute the

i) parametric equations of the line. (4 marks)

ii) symmetric equations of the line. (3 marks)

d. Find an **equation of the plane** that passes through the point (3,8,-5) and is perpendicular to $5\vec{i} + 4\vec{j} - 6\vec{k}$.
(3 marks)

[TOTAL 25 MARKS]

End of page.

APPENDIX

Derivatives: $f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

Differentiation Rules

General Formulae

1. $\dfrac{d}{dx}[f(x)g(x)]=f(x)g'(x)+g(x)f'(x)$
2. $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right]=\dfrac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}$
3. $\dfrac{d}{dx}(x^n)=nx^{n-1}$
4. $\dfrac{d}{dx}[f(u)]=\dfrac{dy}{du}.\dfrac{du}{dx}$

Exponential and Logarithmic Functions

1. $\dfrac{d}{dx}(e^x)=e^x$
2. $\dfrac{d}{dx}(a^x)=a^x\ln a$
3. $\dfrac{d}{dx}(\ln x)=\dfrac{1}{x}$
4. $\dfrac{d}{dx}(\log_a x)=\dfrac{1}{x\ln a}$

Trigonometric Functions

1. $\dfrac{d}{dx}(\sin x)=\cos x$
2. $\dfrac{d}{dx}(\cos x)=-\sin x$
3. $\dfrac{d}{dx}(\tan x)=\sec^2 x$
4. $\dfrac{d}{dx}(\csc x)=-\csc x\cot x$
5. $\dfrac{d}{dx}(\sec x)=\sec x\tan x$
6. $\dfrac{d}{dx}(\cot x)=-\csc^2 x$

Table of Integrals

1. $\int u\,dv=uv-\int v\,du$
2. $\int u^n\,du=\dfrac{u^{n+1}}{n+1}+C,\quad n\neq -1$
3. $\int \dfrac{du}{u}=\ln|u|+C$
4. $\int e^u du=e^u+C$
5. $\int \sin u\,du=-\cos u+C$
6. $\int \cos u\,du=\sin u+C$
7. $\int \sec^2 u\,du=\tan u+C$
8. $\int \csc^2 u\,du=-\cot u+C$
9. $\int \sec u\tan u\,du=\sec u+C$
10. $\int \csc u\cot u\,du=-\csc u+C$

Application of Integrals:

Areas between Curve, $A=\int_a^b[f(x)-g(x)]\,dx$

Differential Equations

Linear Differential Equations

$$\frac{dy}{dx}+p(x)y=q(x) \quad \Rightarrow \quad \mu y=\int \mu q(x)\,dx, \text{ where } \mu=e^{\int p(x)\,dx}$$

Constant Coefficient of Homogeneous Equations

Roots of Auxiliary Equation, $r=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

General Solutions to the Auxiliary Equation:

2 Real & Unequal Roots ($b^2-4ac>0$)	$y=c_1e^{r_1x}+c_2e^{r_2x}$
Repeated Roots ($b^2-4ac=0$)	$y=c_1e^{rx}+c_2xe^{rx}$
2 Complex Roots ($b^2-4ac<0$)	$y=e^{ax}(c_1\cos bx+c_2\sin bx)$

Constant Coefficient of Non-Homogeneous Equations

$y=y_c+y_p$ [y_c: complementary solution, y_p: particular solution]

Vector

Length of Vector

The length of the vector $\mathbf{a}=\langle a_1,a_2,a_3\rangle$ is $|\mathbf{a}|=\sqrt{a_1^2+a_2^2+a_3^2}$.

Dot Product

If θ is the angle between the vector $\mathbf{a}=\langle a_1,a_2,a_3\rangle$ and $\mathbf{b}=\langle b_1,b_2,b_3\rangle$, then

$\mathbf{a}\cdot\mathbf{b}=a_1b_1+a_2b_2+a_3b_3=|\mathbf{a}||\mathbf{b}|\cos\theta$

Cross Product

If θ is the angle between the vector $\mathbf{a}=\langle a_1,a_2,a_3\rangle$ and $\mathbf{b}=\langle b_1,b_2,b_3\rangle$, then

$\mathbf{a}\times\mathbf{b}=\langle a_2b_3-a_3b_2, a_3b_1-a_1b_3, a_1b_2-a_2b_1\rangle$

$|\mathbf{a}\times\mathbf{b}|=|\mathbf{a}||\mathbf{b}|\sin\theta$

Area for parallelogram *PQRS*
$=\left|\vec{PQ}\times\vec{PR}\right|$

Area for triangle PQR
$=\frac{1}{2}\left|\vec{PQ}\times\vec{PR}\right|$

Equation of Lines

Vector equation: $\mathbf{r}=\mathbf{r}_0+t\mathbf{v}$

Parametric equations: $x=x_0+at \qquad y=y_0+bt \qquad z=z_0+ct$

Symmetric equation: $\dfrac{x-x_0}{a}=\dfrac{y-y_0}{b}=\dfrac{z-z_0}{c}$

Equation of Planes

Vector equation: $\mathbf{n}\cdot\mathbf{r}=\mathbf{n}\cdot\mathbf{r}_0$

Scalar equations: $a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

Linear equation: $ax+by+cz+d=0$

Angle between Two Planes: $\theta=\cos^{-1}\left(\dfrac{\mathrm{n}_1\bullet\mathrm{n}_2}{|\mathrm{n}_1||\mathrm{n}_2|}\right)$